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LETTER TO THE EDITOR

COMMENTS ON THE NONLINEAR VIBRATIONS OF CYLINDRICAL SHELLS

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The recent paper by Amabili, Pellicano & Paidoussis (1998) reminds us that the nonlinear vibrations of cylindrical shells continues to be of interest to the research community and offers some subtle issues for study in nonlinear dynamics.

The literature on the topic now spans some thirty years or more. From the present perspective, some of the previously debated issues may be given a clearer explanation and some may even be said to be in some sense largely resolved. It is the purpose of the present note to provide a commentary on the extant literature and to summarize some lessons learned over the years.

Evensen (1963, 1964, 1966, 1967) wrote several pioneering papers on the nonlinear dynamics of rings and cylindrical shells, including both theory and experiment. He noted that for rings the nonlinearity was of softening type and that this was frequently true for cylindrical shells as well. Dowell (1967) confirmed the results of Evensen (1966) for a ring using a somewhat different, but complementary, approach. Dowell & Ventres (1968) then studied the cylindrical shell using the approach they had previously developed for the ring and emphasized that, as the length to radius of the cylindrical shell becomes large, the results for the shell reduce to those for a ring with a softening nonlinearity; while in the limit of small length-to-radius ratios, one finds a hardening nonlinearity, as expected from the previously known results for a flat plate (i.e. a shell of zero curvature or infinite radius).

The difference between the Evensen (1966, 1967) approach and that of Dowell & Ventres (1967) is that, while both use a Galerkin expansion based upon assumed modes, they use different assumed modes. Both assumed a dominant circumferentially asymmetric mode with many node lines around the shell circumference and allow for both in-phase and out-of-phase motion, thus being able to model a circumferential traveling wave which is found in experiments. Also, both included an axisymmetric circumferential mode which experiments and analysis suggest strongly couples to the asymmetric circumferential modes for nonlinear oscillations. Of course, for small, linear oscillations this coupling is weak and tends to vanish.

The principal difference between the analysis of Evensen and that of Dowell for a ring, or the ring limit of a shell, is that Evensen determines the amplitude of the axisymmetric mode in terms of the in-phase and out-of-phase amplitudes of the asymmetric modes by using a kinematic constraint, i.e. the ring is assumed to be inextensional in the circumferential direction, while Dowell allowed the axisymmetric mode to be an independent degree of freedom whose amplitude is determined through the dynamics of the equations of motion.

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Dowell & Ventres (1967) showed that the results of Evensen and Dowell were essentially the same for a ring and in the ring limit for a shell.

For cylindrical shells of intermediate length-to-width ratios, however, the results of Dowell and Evensen do not necessarily agree. The principal reason appears to be the different assumptions made for the axial variation of the axisymmetric mode. Evensen uses an axial variation suggested by his kinematic constraint, while Dowell uses the first axial linear eigenmode associated with the axisymmetric circumferential mode. Thus, for intermediate length-to-radius ratios, Evensen's modal expansion, on the one hand, and that of Dowell & Ventres', on the other, may differ in whether the nonlinearity is softening or hardening. And for the case considered by Varadan *et al.* (1989), according to these authors the two different modal expansions of Evensen and Dowell & Ventres do give different results, i.e. softening versus hardening nonlinearities, respectively.

The way to improve the modal expansion of Dowell & Ventres is clear and straightforward, since the terms in the expansion are simply eigenmodes of the corresponding linear system. And in the recent work by Amabili *et al.* (1998), these authors retain higher axial modes associated with the axisymmetric circumferential mode, while invoking a kinematic constraint of their own to better mimic the mode of Evensen. It seems clear that the most accurate approach would be to construct a higher-dimensional model based upon an expansion in the linear eigenmodes, but that of course complicates the analysis.

The subsequent controversy over several years as discussed by Amabili *et al.* (1998) and others (Prathap 1978; Evensen 1978a, b, Dowell 1978) was a result of the experimental evidence showing agreement with the analysis of Evensen (1967) in displaying a softening nonlinearity for the particular geometry studied, while some analyses based on other modal expansions would show a hardening nonlinearity, as that of Dowell & Ventres (1968). See especially the discussion by Prathap (1978), Evensen (1978a), Dowell (1978) and Evensen (1978b).

This was compounded by some algebraic errors in the analysis of Dowell & Ventres (1968) that were noted by Atluri (1972). These have been corrected recently and have shown the results of Dowell's approach are not essentially changed (Dowell & Ventres 1998). Copies of this corrected paper are available upon request from the author.

Later, Ginsberg (1973) and Chen & Babcock (1975) improved these earlier analyses by considering a perturbation approach, whereby it is assumed the dominant mode is again a single asymmetric mode, but then the contributions of all other modes are treated as a perturbation. This presumably gives the "best" modal expansion subject to the assumption of a perturbation expansion in response amplitude. These authors confirmed a softening nonlinearity for the shell geometries studied.

Perhaps the most important result of the work of all investigators is that, whether the nonlinearity is of a softening or hardening type, for the example treated by most investigators the nonlinearity is extremely weak. For a response amplitude in the dominant asymmetric mode equal to a shell thickness, the linear natural frequency changes by less than 0.5%. Hence the quantitative change due to the nonlinearity is very small, and a perturbation analysis should work well as it indeed appears to do so.

By contrast, one might note that for the hardening nonlinearity typical of a plate, a response amplitude of the order of the plate thickness would change the linear natural frequency by a factor of 2 or 100%, i.e. this nonlinearity is much stronger. For a ring, the nonlinearity is also relatively weaker, although it is possible to achieve frequency changes of several percent with amplitudes of response of several tens of ring thickness (Evensen 1966).

Thus, not surprisingly, in retrospect at least, for shells whose geometry and behavior is intermediate between a ring and a plate the nonlinearity is found to be relatively weaker than the nonlinearities of the softening and hardening type found for a ring and plate, respectively. Furthermore for this weak nonlinearity, apparently small differences in the assumed modal expansion can cause a difference in whether the model gives a softening or hardening nonlinearity.

Again, by retaining more modes in a consistent and complete modal expansion (for example, the linear eigenmodes of small oscillations) one presumably can insure convergence of the modal expansion for large oscillations. This has yet to be done, but is certainly worthy of study and would make a nice complement to the perturbation approach of Ginsberg and Chen & Babcock which does not rely on assumed modes.

Hence, after many years one sees a certain pattern emerging from these several studies, yet some issues and opportunities still remain in studying the subtle behavior of nonlinear vibrations of cylindrical shells. For example, it would be interesting to investigate the limit-cycle oscillations of cylindrical shells due to an aeroelastic instability. The work of Dowell (1969, 1970) for curved plates may be suggestive in that regard, showing much larger limit-cycle oscillations for curved plates relative to those for flat plates. These larger limit-cycle oscillations are consistent with the weaker nonlinearities found due to curvature or shell behavior. Of course the existence of limit-cycle oscillations requires the presence of a hardening nonlinearity for sufficiently large response (though there may be a softening nonlinearity for smaller, but still nonlinear response), and indeed this is found in curved plates (Dowell 1969, 1970).

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